Not elementary integral.
https://www.linkedin.com/feed/update/urn:li:activity:6643140893744467968
Evaluate

$$\int_0^1 \frac{x^2 e^x}{(x+1)^2}dx$$

Solution by Arkady Alt, San Jose ,California, USA.

$\int_0^1 \frac{x^2 e^x}{(x+1)^2}dx = \frac{1}{e}\int_0^1 \frac{\left((x+1)^2 - 2(x+1) + 1\right)e^{x+1}}{(x+1)^2}dx =$

$\frac{1}{e}\left(\int_0^1 e^{x+1}dx - 2\int_0^1 \frac{e^{x+1}}{x+1}dx + \int_0^1 \frac{e^{x+1}}{(x+1)^2}dx\right) =$

$\frac{1}{e}\left(e(e-1) - 2\int_1^2 \frac{e^x}{x}dx + \int_1^2 \frac{e^x}{x^2}dx\right).$

Since $\int_1^2 \frac{e^x}{x^2}dx = \begin{bmatrix} u' = \frac{1}{x^2}; u = -\frac{1}{x} \\ v = e^x; v' = e^x \end{bmatrix} = \left(-\frac{e^x}{x}\right)_1^2 + \int_1^2 \frac{e^x}{x}dx =$

$e - \frac{1}{2}e^2 + \int_1^2 \frac{e^x}{x}dx$ then $\int_0^1 \frac{x^2 e^x}{(x+1)^2}dx = \frac{1}{e}\left(e(e-1) + e - \frac{1}{2}e^2 - \int_1^2 \frac{e^x}{x}dx\right) =$

$\frac{1}{e}\left(e(e-1) + e - \frac{1}{2}e^2 - \int_1^2 \frac{e^x}{x}dx\right) = \frac{1}{2}e - \frac{1}{e}\int_1^2 \frac{e^x}{x}dx.$

Thus, remains calculate $\int_1^2 \frac{e^x}{x}dx.$

We have $\int_1^2 \frac{e^x}{x}dx = \int_1^2\left(\frac{1}{x} + \sum_{n=1}^{\infty}\frac{x^{n-1}}{n!}\right)dx = \ln 2 + \sum_{n=1}^{\infty}\frac{1}{n!}\int_1^2 x^{n-1}dx =$

$\ln 2 + \sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{x^n}{n}\right)_1^2 = \ln 2 + \sum_{n=1}^{\infty}\frac{2^n - 1}{n \cdot n!}$ and, therefore,

$$\int_0^1 \frac{x^2 e^x}{(x+1)^2}dx = \frac{1}{2}e - \frac{1}{e}\ln 2 - \frac{1}{e}\sum_{n=1}^{\infty}\frac{2^n - 1}{n \cdot n!}.$$